(Time: 3 hours)   Max.Marks:80

N.B   (1) Question No.1 is compulsory
        (2) Answer any three questions from Q.2 to Q.6
        (3) Use of Statistical Tables permitted
        (4) Figures to the right indicate full marks.

1
a) Prove that $\sec h^{-1}(\sin \theta) = \log\left(\cot \dfrac{\theta}{2}\right)$ 5

b) If $z = x^y + y^x$ then prove that $\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{\partial^2 z}{\partial y \partial x}$ 5

c) If α, β are the roots of the quadratic equation $x^2 - 2\sqrt{3}\, x + 4 = 0$,   5
    find the value of $\alpha^3 + \beta^3$

d) Test the consistency and if possible solve   5
    $2x - 3y + 7z = 5,\ 3x + y - 3z = 13,\ 2x + 19y - 47z = 32$

2
a) Is $A = \begin{bmatrix} \dfrac{2+i}{3} & \dfrac{2i}{3} \\ \dfrac{2i}{3} & \dfrac{2-i}{3} \end{bmatrix}$ a unitary matrix?   6

b) Find the nth derivative of $y = \dfrac{4x}{(x-1)^2(x+1)}$ 6

c) If $u = \dfrac{x^4 + y^4}{x^2 y^2}$ then find the value of   8

    $x \dfrac{\partial u}{\partial x} + y \dfrac{\partial u}{\partial y} + x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy \dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2}$ at $x = 1$ and $y = 2$

3 a) Prove that $\log(1 + \cos 2\theta + i \sin 2\theta) = \log(2\cos\theta) + i\theta$ 6

   b) Solve $x^7 + x^4 + i(x^3 + 1) = 0$ using De Moivre's theorem   6

   c) Discuss for all values of k for which the system of equations   8
       has a non-trivial solution
       $2x + 3ky + (3k + 4)z = 0,$
       $x + (k + 4)y + (4k + 2)z = 0,\ x + 2(k + 1)y + (3k + 4)z = 0$

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4 a) If $u = \log(r)$ and $r = x^3 + y^3 - x^2 y - xy^2$ then show that   6

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$$

b) Find two non-singular matrices P and Q such that PAQ is in   6

the normal form where $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$

c) Prove that $\tan^{-1}(e^{i\theta}) = \frac{n\pi}{2} + \frac{\pi}{4} - \frac{i}{2} \log \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$ 8

5 a) Considering principal value, express in the form $a + ib$ the   6
quantity $(\sqrt{i})^{\sqrt{i}}$

b) Prove that $\tan 5\theta = \dfrac{5 \tan \theta - 10 \tan^3 \theta + \tan^5 \theta}{1 - 10 \tan^2 \theta + 5 \tan^4 \theta}$ 6

c) If $y = e^{a \sin^{-1} x}$, then Prove that   8
$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$ Also find $y_n(0)$

6 a) If $u = \dfrac{1}{r}$, $r = \sqrt{x^2 + y^2 + z^2}$ then prove that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0$ 6

b) If $\dfrac{3}{x} + \dfrac{4}{y} + \dfrac{5}{z} = 6$ find the values of x, y, z such that $x + y + z$ is   6
minimum

c) Prove that every Skew-Hermitian matrix can be expressed in   8
the form B+iC, where B is real Skew-Symmetric and C is real
Symmetric matrix and express the matrix

$A = \begin{bmatrix} 2i & 2+i & 1-i \\ -2+i & -i & 3i \\ -1-i & 3i & 0 \end{bmatrix}$ as B+iC where B is real Skew-

symmetric matrix and C is real Symmetric matrix

(2 Hours) [Total Marks: 60]

N.B.: (1) Question No. 1 is compulsory.

(2) Attempt any three questions from Q.2 to Q.6.

(3) Assume suitable data wherever required.

(4) Figures to the right indicate marks.

Q1. Attempt any five [15mks]

a. Draw the following planes in a cubic unit cell (121), (100), (011).

b. The diameter of 5th dark ring in Newton's ring experiment was found to be 0.42 cm. Determine the diameter of 10th dark ring in the same set up.

c. An electron is bound in a one-dimensional potential well of width 2 A° but of infinite height. Find its energy values in the ground state and in first excited state.

d. Define superconductivity and explain the terms critical temperature and critical magnetic field.

e. Find the resistivity of intrinsic germanium at 300 K. Given density of carriers is 2.5×10^{19} /m^3, mobility of electrons is 0.39 m^2/volt-sec and mobility of holes is 0.19 m^2/volt-sec.

f. What are matter Waves? State three properties of matter waves.

g. Explain the formation of colours in thin film.

Q2 a) State Hall Effect. Obtain an expression for Hall voltage. Calculate the mobility of charge carriers in a doped Si, whose conductivity is 100 per ohm meter and Hall coefficient is 3.6×10^{-4} m^3/C. [8mks]

b) Obtain an expression for Optical Path Difference in a thin film of uniform thickness observed in reflected light. Hence obtain conditions for maxima and minima. [7mks]

Q3a) Explain with neat diagram the effect of doping and temperature on the fermi level in N type extrinsic semiconductor. What is the probability of an electron being thermally excited to the conduction band in Si at 20° C. The band gap energy is 1.12 eV [8mks]

b) Show that the energy of an electron in a one-dimensional deep potential well of infinite height varies as the square of the natural numbers. [7mks]

Q4.a) Explain Bragg's spectrometer for the investigation of crystal structure with the help of a neat diagram. [5mks]

b) Derive one dimensional Schrödinger's time dependent equation for matter waves. [5mks]

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c). White light is incident on a soap film at an angle $\sin^{-}(4/5)$ and the reflected light is observed with a spectroscope. It is found that two consecutive dark bands correspond to wavelength 6100 A° and 6000 A°. If the refractive index of the film is 4/3, calculate its thickness. [5mks]

Q5 a) Find the de Broglie wavelength of (i) an electron accelerated through a potential difference of 182 Volts and (ii) 1 Kg object moving with a speed of 1 m/s. Comparing the results, explain why is the wave nature of matter not apparent in daily observations? [5mks]

b). Derive an expression for interplanar spacing in a cubic unit cell? [5mks]

c) Explain the principle and working of Supercapacitors? [5mks]

Q6a) Explain principle, construction and working of Light Emitting Diode? [5mks]

b). State Meissner's effect. Show that superconductors exhibit perfect diamagnetism [5mks]

c). We wish to coat a flat slab of glass with refractive index 1.5 with a thinnest possible film of transparent material so that light of wavelength 600 nm incident normally is not reflected. We have two materials to choose from M_1 (μ= 1.21) and M_2 (μ =1.6). Which one would be appropriate? What will be the minimum thickness of coating? [5mks]

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Time: - 3 hr.   **Maximum Marks: 80**
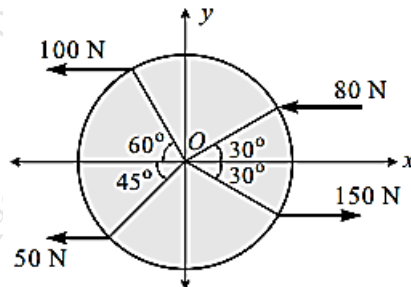
N.B.: (1) Question No. 1 is compulsory.

(2) Attempt any three questions from the remaining five questions.

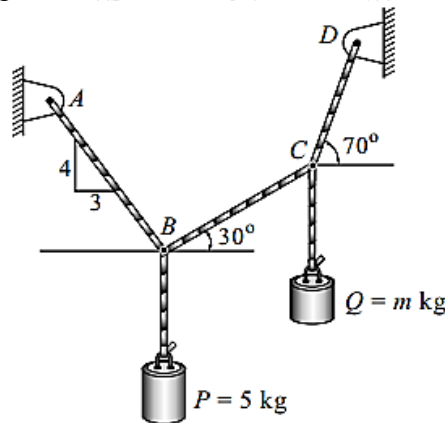(3) Assume suitable data if necessary and mention the same clearly.

(4) Take g=9.81 m/s^2

Q.1   Attempt any five.

a. For the force system shown. Find the resultant and locate it with respect to   [4]
    O if the radius of plate is 1m.

b. For the system shown in fig. Determine mass m to maintain the equilibrium.   [4]

c. Define laws of friction.   [4]

d. A rectangular plate weighing 500 N is suspended in the horizontal plane using   [4]
    three cables. Find the tension in each cable.
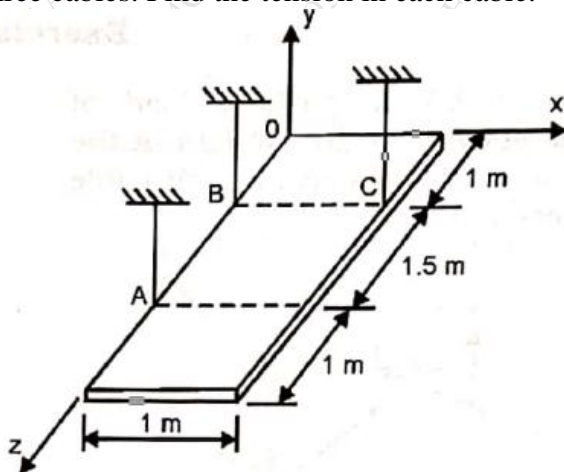
e. The acceleration of the particle is given by the equation a = -0.05v^2 m/s^2 [4]
    where, v is the velocity in m/s and x is the displacement in m. Knowing at
    v=20 m/s at x=0 determine (i) the position of the particle at v= 15 m/s (ii)
    acceleration at x=50 m.

f. Define General plane motion and ICR. What are the properties of an ICR.   [4]
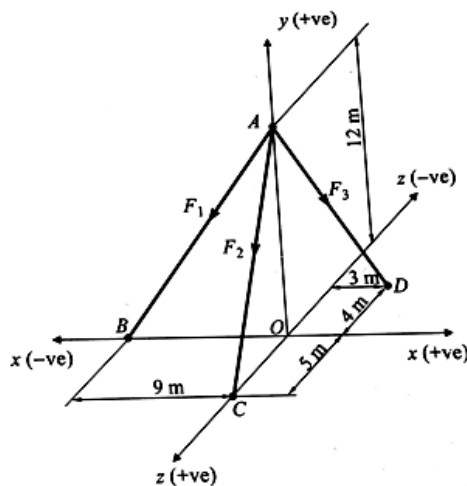
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Q.2 a. Find the minimum force P required to pull the block. Take the coefficient of friction between A and B as 0.3 and between B and floor as 0.25. [10]
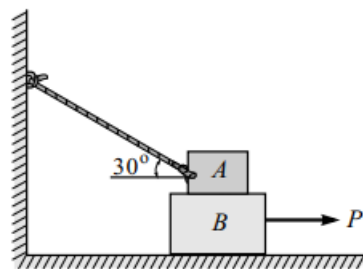

b. For given system find resultant and its point of application with respect to point A. [6]
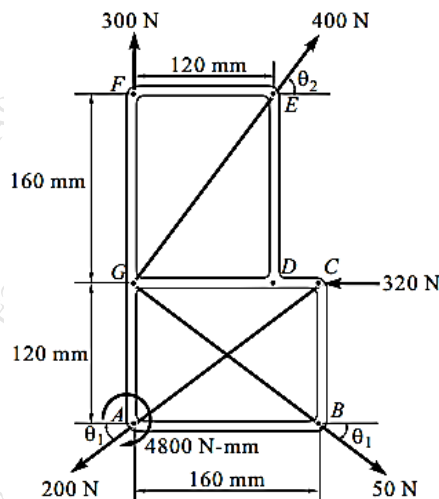
c. The resultant of the three concurrent space forces at A is R= - 788j N. Find magnitude of F_1, F_2, F_3 forces. [4]


Q.3 a. Two blocks W_1 and W_2 connected by a horizontal bar AB are supported on rough planes as shown in fig. Considering the coefficient of friction between block A and ground as 0.4 and angle of friction for block B is 20^0. Find the smallest weight $W_1$for which the equilibrium can exist, if W_2 is 2250 N [8]
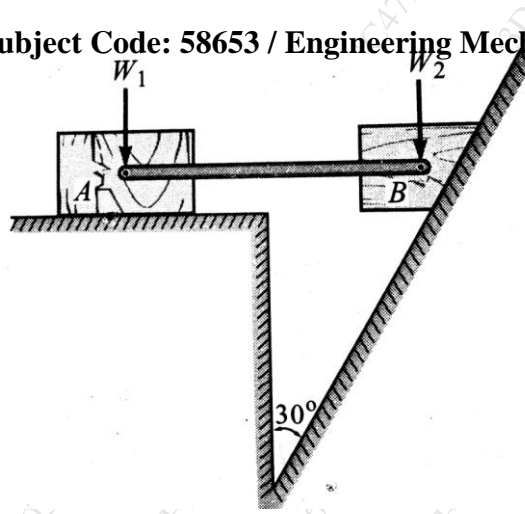
b. For the system shown in fig. if the collar is moving upwards with a velocity [8]
   of 1.5m/s. Locate the ICR for the instant shown. Determine angular velocity
   of rod AB, Velocity of A and velocity at the midpoint of AB.
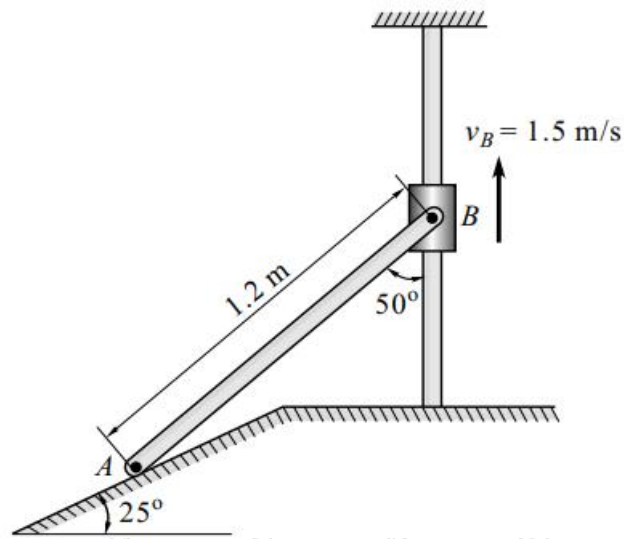
c. A ball thrown with a speed of 12m/s at an angle of 60^0 with a building strikes [4]
   the ground 11.3m horizontally from the foot of the building as shown in fig.
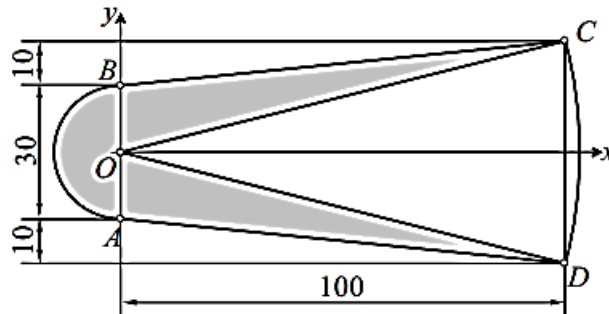   Determine the height of the building.
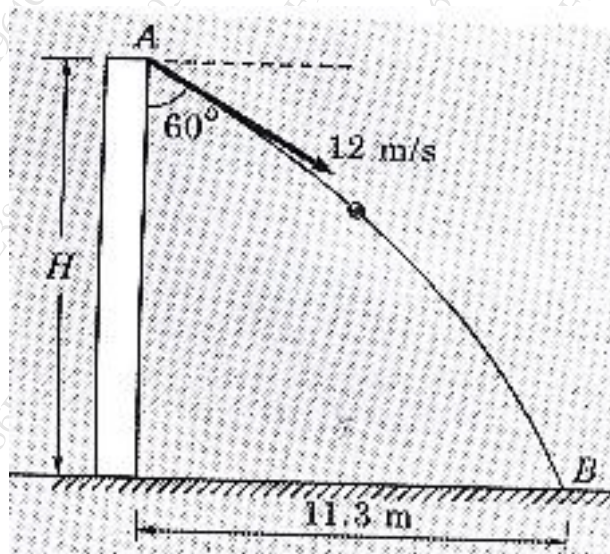
Q.4 a. A car moves along a straight road such that its acceleration time motion is [8] described by the graph shown in fig. construct v-t and s-t graphs and determine the maximum speed and maximum distance covered.
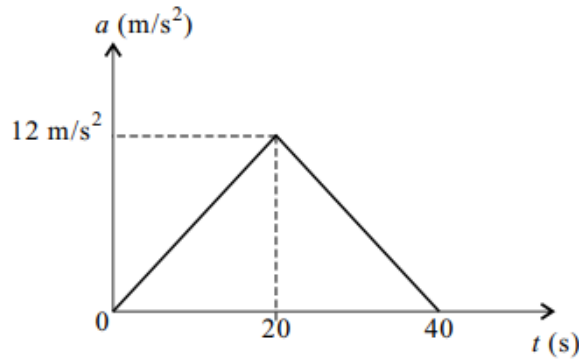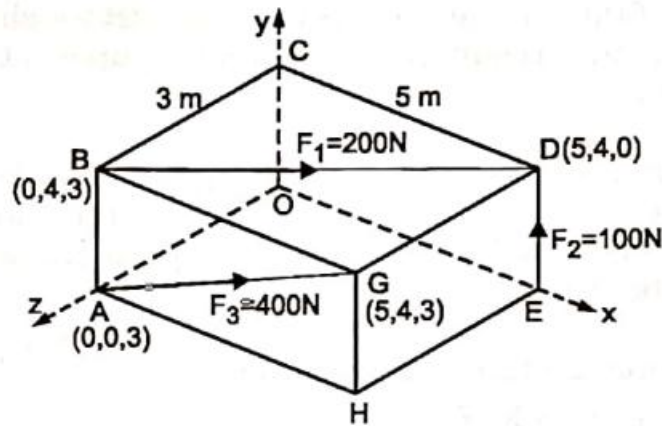

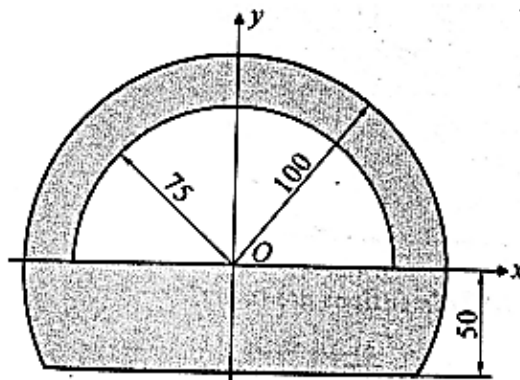Determine the centroid of the shaded area.

b.

[8]

c. A rectangular parallelepiped carries four forces shown in fig. Reduce the force [4] system to a resultant force applied at the origin.
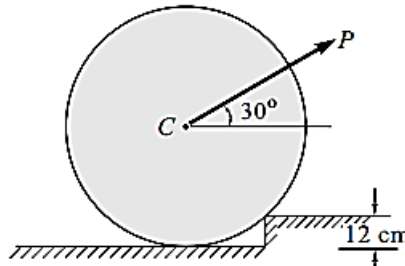

Q.5 a. Find the centroid of the shaded area. [8]

[6]

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b. Determine force P applied at 45^0 to the horizontal just necessary to start a roller of 100 cm diameter and weighing 100 kg over a block of 12 cm high.


c. A point moving along a path $y=x^2/3$ with a constant speed of 8m/s. What are the x and y components of its velocity when x=3m? Also, find the radius of curvature and acceleration. [6]

Q.6 a. Knowing that at the instant the angular velocity of rod BE is 4 rad/sec counterclockwise determine the angular velocity of rod AD and velocity of collar D. [8]
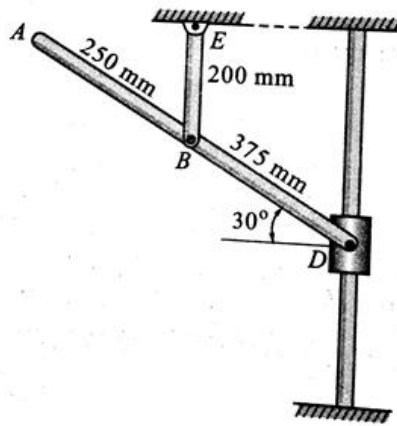
b. Find the support reactions for the beam loaded as shown in fig. [6]
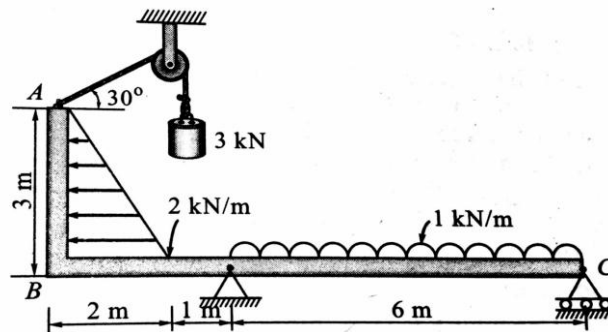
c. Two identical rollers of mass 50kg are supported as shown in figure. To maintain the equilibrium, Determine the support reactions assuming all smooth surfaces. [6]


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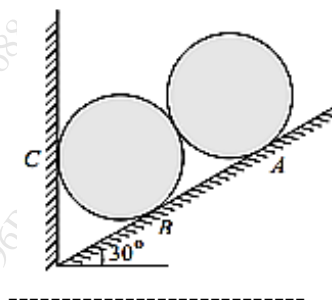
Time: 3 Hr   **Max. Marks:- 80**

Note: - 1) Question No1. is compulsory.
     2) Attempt any Three out of remaining.

Q1   Attempt **any FOUR**   **(20)**

    a)   Derive an emf equation of a DC Machine   **(5)**

    b)   Calculate the total power and readings of two wattmeters connected to measure   **(5)**
        power in three phase balanced load if the apparent power is 15KVA & load pf
        is 0.8 lag.

    c)   In inductive coil containing resistance 20Ω and inductance of 0.2H is connected   **(5)**
        in parallel with a capacitor of 150µF. Find resonant frequency of the circuit,
        dynamic impedance of the circuit and current in the circuit.

    d)   Find current in 5Ω by nodal analysis.   **(5)**
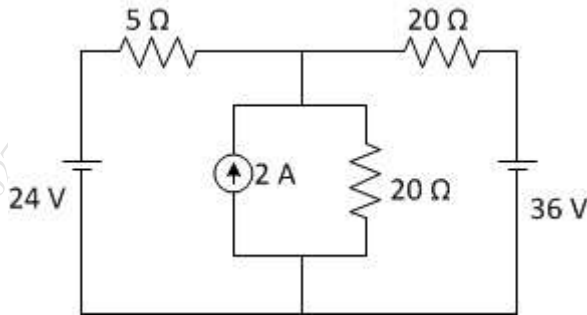
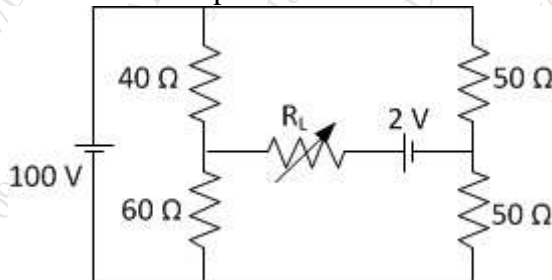   **e)**   Three currents are meeting at a point. Find the resultant current.   **(5)**
        $i1 = 50 \sin(\omega t)$A, $i2 = 25 \cos(\omega t - \frac{\pi}{6})$A,
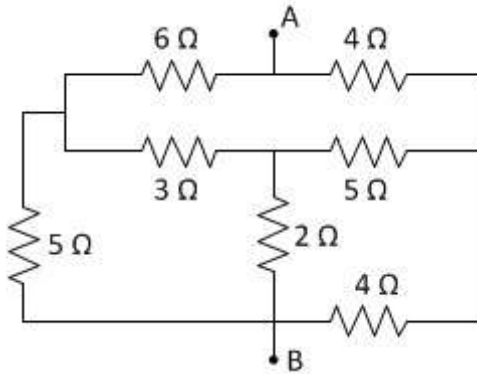 $i3 = -10 \sin(\omega t + \frac{\pi}{4})$A.

Q2 **a)**   Find current in 1Ω resistance by superposition theorem.   **(10)**
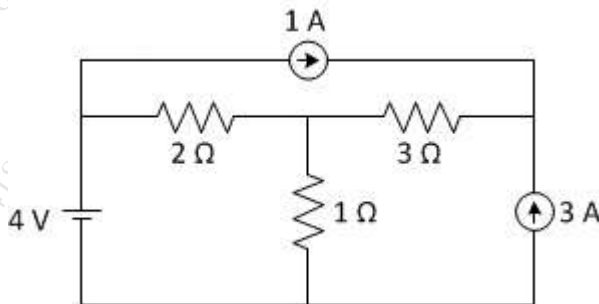
    b)   Find maximum power in RL.   **(10)**

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Q3 **a)** Find equivalent resistance between A & B in the network shown. **(10)**

b) A 100Ω resistance is connected in series with a choke coil. A voltage of 400V, 50Hz is applied across this combination. The voltage across resistance and coil is 200V and 300V respectively. Find resistance and reactance of a coil, power factor of a coil and complete circuit, power absorbed by resistor, coil and complete circuit. **(10)**

Q4 **a)** A resistor and a capacitor are in series with a variable inductor. When the circuit is connected to a 220V, 50Hz supply, the maximum current obtainable by varying the inductance is 0.314A. The voltage across capacitor is then 800V, find R, L and C. **(10)**

b) Explain working principle of three phase induction motor and mention its types. **(5)**

c) Explain working of permanent magnet stepper motor. **(5)**

Q5 **a)** Derive relation between line & phase voltage and line & phase current in three phase STAR connected circuit. Also derive equation of active, reactive, and apparent power. **(10)**

b) A balanced 3 phase star connected load of 100kW takes a leading current of 80A, when connected across 3 phase 1100V, 50Hz supply. Find the circuit constants of the load per phase, power factor of the load. **(10)**

Q6 **a)** A pure resistor R, a choke coil and a pure capacitor of 15.91microfarad are connected in series across a supply of V volts and carries a current of 0.25A. The voltage across the choke coil is 40V, the voltage across the capacitor is 50V and the voltage across the resistor is 20V. The voltage across the combination of R and the choke coil is 45V. Calculate i) supply voltage ii) frequency iii) the power in choke coil. **(10)**

b) An impedance Z1 is connected in series with two parallel impedances Z2 and Z3. Z1 consists of a resistance of 6 ohm in series with an inductor of 0.01H. Z2 consists of a resistance of 4 ohm in series with an inductor of 0.02H. Z3 consists of a resistance of 2 ohm in series with a capacitor of 200 microfarad. A voltage of 100V at 50Hz frequency is applied across complete circuit. Find impedance of the circuit, current drawn by the circuit, power absorbed by all the three impedances and by complete circuit. **(10)**

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<div align="center">

Time: 2 hours   **Maximum marks: 60**
</div>

N.B.   1. Question **Number 1 is compulsory**

        2. Attempt **any three** questions from **Q.2 to Q.6**

        3. Draw neat labeled diagrams where necessary.

        4. Write chemical reactions where required.

        5. Figures to right indicate full marks.

Additional Data: H =1, C = 12, N =14, O =16, Na =23, Mg =24, S = 32, Cl =35.5,K =39, Ca = 40

Q.1]   Attempt <u>**any five**</u> from the following:-   **15**
  (a)   Write a brief note on Ultrafiltration.

  (b)   Write synthesis, properties and uses of PMMA polymer.

  (c)   Write basic postulates of molecular orbital theory.

  (d)   Explain the term 'Viscoelasticity' with reference to the polymers.

  (e)   A hard water sample was found to have following salts: $Ca(HCO_3)_2$ = 132 mg/L, $MgSO_4$ = 164mg/L, $CaCl_2$ = 127 mg/L, NaCl = 98mg/L. Calculate temporary, permanent and total hardness of the given sample of water.

  (f)   Define the terms involved Gibbs phase rule equation.

  (g)   Is cyclopentadiene molecule aromatic in the nature? Why?

Q.2] (a) A standard hard water sample was prepared by dissolving 1.8 g $CaCO_3$ in   **6**
           1.5L of distilled water. 50 ml of this water required 46 ml of EDTA. 50 ml of unknown hard water required 36 ml of same EDTA. This unknown hard water was boiled, cooled and filtered. 50 ml of this filtered water required 29 ml of same EDTA, Calculate temporary, permanent and total hardness of unknown hard water sample.

  (b)   Describe bonding in benzene molecule and explain why it is aromatic?   **5**

  (c)   What is the role played by Plasticizer and Catalyst in making of plastic?   **4**

Q.3] (a) Draw a neat labeled diagram and explain Extrusion Molding of plastic.   **6**

  (b)   Draw a neat labeled phase diagram of Pb –Ag system and explain the   **5**
           validity of reduce phase rule equation.

  (c)   Draw a suitable Molecular Orbital Diagram and explain why Be_2 does not   **4**
           exist as a molecule?

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Q.4] (a) Write a short note on 'Ion Exchange method' of demineralization of the hard water.   **6**

(b) Draw a neat labeled molecular orbital diagram of CO (Carbon monoxide) molecule and explain its bond order and magnetic property.   **5**

(c) Explain bonding in pyrrole molecule.   **4**

Q.5] (a) Draw a neat phase diagram of one component water system and explain applicability of Gibbs Phase Rule.   **6**

(b) A sample of the polymer contains 50 molecules of molecular weight 10000 units each, 30 molecules each with molecular weight of 30000 and 20 molecules each having molecular weight of 60000 units. Calculate number averaged and weight averaged molecular weights of polymer and Poly-dispersity index (PDI).   **5**

(c) Define COD. In determination of COD, 25 ml of sewage water was refluxed with 25 ml of 0.5N of $K_2Cr_2O_7$ in acidic medium. The unreacted $K_2Cr_2O_7$ required 19 ml of 0.5N FAS (Ferrous ammonium sulphate). In the blank determination, 25 ml of distilled water was refluxed with 25 ml of 0.5N of $K_2Cr_2O_7$ in acidic medium which required 24 of 0.5N FAS. Calculate COD.   **4**

Q.6] (a) Write a brief note on 'Conducting Polymers'   **6**

(b) Draw a molecular orbital diagram of O_2 (oxygen) molecule and explain its bond order and magnetic property.   **5**

(c) Write Merits and Demerits of Phase Rule.   **4**