Time: 3 Hours

Total Marks:80

Instructions:

1) Question 1 is compulsory
2) Attempt any three from the remaining questions.

1-a) Prove that the matrix $\frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary. (5 Marks)

1-b) State Euler's theorem on homogeneous function of two variables and
evaluate $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ where, $u = \frac{x+y}{x^2+y^2}$. (5 Marks)

1-c) Separate into real and imaginary part of $\cos^{-1}\left(\frac{3i}{4}\right)$. (5 Marks)

1-d) If $y = 2^x \sin^2 x \cos x$ find y_n. (5 Marks)

2-a) Show that $\frac{\sin 5\theta}{\sin\theta} = 16\cos^4\theta - 12\cos^2\theta + 1$ (6 Marks)

2-b) If $u = \tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ P.T $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -2\sin^3 u \cos u$ (6 Marks)

2-c) Test for consistency the following system & solve them if consistent. (8 Marks)
$$x_1 - 2x_2 + x_3 - x_4 = 2$$
$$x_1 + 2x_2 + 2x_4 = 1$$
$$4x_2 - x_3 + 3x_4 = -1$$

3-a) Show that minimum value of $u = xy + a^3\left(\frac{1}{x} + \frac{1}{y}\right)$ is $3a^2$. (6 Marks)

3-b) Using Newton-Raphson method find the root of equation $2x^3 - 3x + 4 = 0$ lying between
-2 and -1 correct to four places of decimals. (6 Marks)

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3-c) If $y^{1/m} + y^{-1/m} = 2x$ prove that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$. (8 Marks)

4-a) Solve $x^5 = 1 + i$ and find the continued product of the roots. (6 Marks)

4-b) Apply Gauss elimination method to solve the equations x+3y-2z=5, 2x+y-3z=1, 3x+2y-z=6. (6 Marks)

4-c) For what value of λ the equations $x + 2y + z = 3, x + y + z = \lambda, 3x + y + 3z = \lambda^2$ have a solution and solve them completely in each case. (8 Marks)

5-a) Evaluate $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$. (6 Marks)

5-b) If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, then show that $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$. (6 Marks)

5-c) Prove that $\log\left[\frac{sinx+iy}{sinx-iy}\right] = 2i \tan^{-1}(cotx \ tanhy)$ (8 Marks)

6-a) Find the nth derivative of $\frac{x}{(x-1)(x-2)(x-3)}$ (6 Marks)

6-b) Reduce the following matrix to its normal form and hence find its rank.

$$A = \begin{bmatrix} 3 & -2 & 0 & 1 \\ 0 & 2 & 2 & 7 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

(6 Marks)

6-c) i) Express $(2x^3 + 3x^2 - 8x + 7)$ in terms of $(x - 2)$ using Taylor's theorem.

ii) Prove that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} + \dots$ (8 Marks)

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(2 Hours)   **[Total Marks:60]**

N.B.:   (1) Question. **1** is **compulsory.**
      (2) Attempt any **three** questions from the remaining questions N0.**2** to **6.**
      (3) **Assume** suitable **data** wherever required.
      (4) **Figures** to the **right** indicate **marks.**

1. Attempt any **five** questions from the following-   **15**
     (a) Calculate atomic packing fraction of FCC Lattice.
     (b) Derive the De-Broglie wavelength.
     (c) Draw the energy band diagram for p-n junction diode in equilibrium condition.
     (d) Define: Persistent current, critical temperature, critical magnetic field.
     (e) What is reverberation time? Explain its formula.
     (f) With the help of diagram state direct and inverse piezoelectric effect.
     (g) The resistivity of intrinsic material at room temperature is 2×10^{-4} Ohm-cm. If the mobility of electron is $6 \ m^2/V\text{-sec}$ and mobility of hole is $0.2 \ m^2/V\text{-sec}$. Calculate its intrinsic carrier density.

2. (a)   Prove that electron cannot survive within the nucleus.   **8**
       An electron has a speed of 400m/s with uncertainty of 0.01% . Find the accuracy in its position.
   (b) What is the Hall effect? obtain the expression for (a) Hall voltage and (b) Hall Coefficient.   **7**

3. (a) With neat diagram of unit cell explain the structure of diamond crystal.   **8**
   (b) Explain variation of Fermi level with temperature in n-type semiconductor.   **7**
      What is the probability of an electron being thermally excited to the conduction band? If the Si is at 27^0C. The band gap energy is 1.12eV ($k=1.38 \times 10^{-23}$ J/K)

4. (a) Distinguish between Type I and Type II superconductors.   **5**
   (b) A class room has dimension of $(20 \times 15 \times 10) \ m^{3;}$ the reverberation time is 3 sec.   **5**
      Calculate the total absorption of its surfaces and average coefficient of absorption
   (c) How ultrasonic waves are produced using quartz crystal in an oscillator?   **5**

5. (a) Show that for an intrinsic semiconductor, the Fermi level lies half way between   **5**
      Conduction and valence band..
   (b) Find the depth of sea water from a ship on the sea surface if the time interval of 2 seconds is required to receive the signal back.
      Given that temperature of sea water $=20^o$C , Salinity of sea water $=10$gm/lit   **5**
   (c) The lowest energy of an electron trapped in a one dimensional box is   **5**
      3.2×10^{-18} J. Calculate the width of the box. Also calculate the next two energies in eV the particle can have?

6. (a) Define ligancy and critical radius ratio. Calculate critical radius ratio for ligancy 6. **5**
   (b) Obtain one dimensional time dependent Schrodinger equation   **5**
   (c) Explain photovoltaic effect and write a note on solar cell.   **5**

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